1. **Overview Statement**: Briefly summarize the assessment activities that were undertaken this academic year, indicating:
   
a. which program learning outcomes were assessed this year.

b. who in your department/program was involved in the assessment of the above learning outcomes

This spring the mathematics department assessed the following three outcomes:

- **(Outcome 1)** Use techniques of differentiation and integration of one and several variables.
- **(Outcome 2)** Use differentiation and integration to solve problems in mathematics and other disciplines.
- **(Outcome 4)** Give direct proofs, proofs by contradiction and proofs by induction.

The first outcome was assessed by Professors Stephen Devlin, Tristan Needham and Paul Zeitz. 
The second outcome was also assessed by Professors Devlin, Needham and Zeitz, and the fourth outcome was assessed by Professors Devlin and Zeitz.

2. **Please Answers the Following Questions for Each of the Student Outcomes Assessed:**
   
a. **What did you do?**
   Describe clearly and concisely how you assessed the learning outcomes that were evaluated this year (e.g., measures, research methods, etc.). [please use bullet points to answer this question]

   We selected representative questions from the final exams in the following classes:

   - **Outcome 1**: Math 109 Calculus I, Math 110 Calculus II, and Math 370
Probability and Statistics
- Outcome 2: Calculus I, Calculus II, and Probability and Statistics

The students’ responses to the various questions were rated as exemplary, acceptable, or unacceptable, and the responses were tabulated.

b. **What did the faculty in the department or program learn?**
   Summarize your findings and conclusions as a result of the assessment indicating strengths and weaknesses in student learning demonstrated by this assessment.

   See the included report “Findings of the Mathematics Department Assessment of Learning Outcomes.” From the findings it appears that the department is doing reasonably well teaching Outcomes 1 and 4, but Outcome 2 may need additional attention.

c. **What will be done differently as a result of what was learned?**
   Discuss how courses and/or curricula will be changed to improve student learning as a result of the assessment. Include a discussion of how the faculty will help students overcome their weaknesses and improve their strengths.

   The results of the assessment only became available after the end of the Spring 2009 Semester. The department will discuss the outcomes and possible changes in the Fall 2009 Semester.

3. **Attach a copy of the components of the department/program assessment plan that have been modified since its initial submission:**
   a. Program Mission
   b. Program Learning Goals
   c. Program Learning Outcomes
   d. Program Learning Rubrics aligned with outcomes
   e. Curriculum map that shows the courses that pertain to the outcome

   The only changes have been minor: correction of typographical errors, changes in wording to clarify several passages, and elimination of text related to the preliminary nature of the plan.

   **Please return to: Provost Office by June 1, 2009**

   You can send your replies as either a Word attachment (to: marin@usfca.edu) or as a hard copy to: Provost Office, Lone Mountain Rossi Wing 4th floor.
If you have any questions, please contact: William Murry, Director of Institutional Assessment (wmurry@usfca.edu or x5486).
Findings of the Mathematics Department Assessment of Learning Outcomes

Department of Mathematics
University of San Francisco
San Francisco, CA 94117

June 1, 2009

1 Introduction

This is a summary of the findings of the Mathematics Department in its assessment of Learning Outcomes 1, 2 and 4. It is supporting material for the response to Question 2b in the “Program Assessment Report” for the academic year 2008–2009. All of the outcomes were assessed by selecting representative questions from final exams of classes offered in the spring semester of 2009. A class was chosen if one or more of the three outcomes received significant coverage in the clas.

The three outcomes assessed this year were

1. Use techniques of differentiation and integration of one and several variables.
2. Use differentiation and integration to solve problems in mathematics and other disciplines.
3. Give direct proofs, proofs by contradiction and proofs by induction.
2 The Questions and the Results

1. Math 109, Calculus I, Prof. Devlin. For each function below, find the derivative. You don’t need to simplify your answers.

   (a) \( e^x \sqrt{x^2 + 1} \)
   (b) \( \frac{x - \sqrt{x}}{1 + x^3} \)
   (c) \( \sqrt{\sin \sqrt{2x}} \)
   (d) \( \frac{\ln(x)}{x^2} \)

   Used in assessing outcome 1.

2. Math 109, Calculus I, Prof. Devlin.

   (a) Use implicit differentiation to find \( \frac{dy}{dx} \) where \( y \) is given implicitly by the equation \( x^3 + y^3 = 6xy \).
   (b) Find the line tangent to the curve defined in (a) at the point (3, 3).

   Used in assessing outcomes 1 and 2.

3. Math 109, Calculus I, Prof. Devlin. A paper cup has the shape of a cone (with the point at the bottom) with height 10 cm and radius 3 cm at the top. If water is poured into the cup at a constant rate of 2 \( \text{cm}^3/\text{s} \), how fast is the water level in the cup rising when the water is 5 cm deep?

   Used in assessing outcomes 1 and 2.

4. Math 109, Calculus I, Prof. Devlin. Find the local and absolute extreme values of the function \( f(x) = x\sqrt{1-x} \) on the interval \([0, 1]\).

   Used in assessing outcomes 1 and 2.
5. Math 109, Calculus I, Prof. Devlin. A rectangular storage container with an open top is to have a fixed volume of 10 m$^3$. The length of the base is three times the width. Material for the base costs $5 per square meter. Since the sides are to be made out of decorative material, they cost $12 per square meter. Find the dimensions of the box that will minimize the cost of materials.

Used in assessing outcomes 1 and 2.

6. Math 109, Calculus I, Prof. Devlin. Compute the following integrals:

(a) \( \int \frac{x}{x^2+1} \, dx \)

(b) \( \int \sqrt{x} \sin(1 + x^2) \, dx \)

(c) \( \int_0^1 x^2(1 + 2x^3)^5 \, dx \)

Used in assessing outcome 1.

7. Math 110, Calculus II, Prof. Needham. Evaluate the indefinite integral, \( \int \sin^2 x \cos^3 x \, dx \).

Used in assessing outcome 1.

8. Math 110, Calculus II, Prof. Needham. Use partial fractions to evaluate the indefinite integral, simplifying your answer into a single logarithm:

\[ \int \frac{x - 1}{x^2 + 3x + 2} \, dx. \]

Used in assessing outcome 1.


(a) Prove that \( \mathbb{Q} \) is closed under subtraction. You may use the fact that \( \mathbb{Z} \) is closed under addition, subtraction, and multiplication.

(b) Prove that if \( r \) is rational and \( \alpha \) is irrational, then \( r + \alpha \) is irrational.
10. Math 235, Intro to Formal Methods, Prof. Zeitz.

(a) Prove, using mathematical induction, that for all positive integers \( n \), \( 3^n > 2n \).

(b) Let the Fibonacci sequence be defined by \( f_1 = f_2 = 1 \) and \( f_{k+2} = f_{k+1} + f_k \), for \( k \geq 1 \). The first few terms are 1, 1, 2, 3, 5, 8, … Prove that for all positive integers \( n \), \( f_{3n} \) is even. You may use induction, or any other method.

11. Math 235, Intro to Formal Methods, Prof. Zeitz. Prove that in any group, the identity element is unique. You may use any of the four group axioms (closure, associativity, existence of an identity element, existence of inverses), but you may not use any other facts or theorems about groups.

12. Math 370, Intro to Formal Methods, Prof. Zeitz.

(a) Let \( X \) be a continuous random variable with a PDF defined by \( f(x) = e^{-x} \) if \( x \geq 0 \) and \( f(x) = 0 \) if \( x < 0 \). Compute the moment-generating function \( M_X(t) \).

(b) Let \( Y \) be the random variable obtained by taking three independent outputs from \( X \) and adding them. In other words, imagine that \( X \) is a machine that outputs a number when you press a button. You press the button three times, record the outputs, and add them. That is \( Y \). Find \( M_Y(t) \).

(c) Let \( W \) be the random variable obtained by taking three independent outputs from \( X \) and computing their average. Find \( M_W(t) \).
13. *Math 435, Modern Algebra, Prof. Devlin*. Show that the set $\mathbb{Z}$ of integers with the operation $\star$ defined for all $a, b \in \mathbb{Z}$ by $a \star b = a + b - 1$, is a group.

Used in assessing outcome 4.


14. *Math 435, Modern Algebra, Prof. Devlin*. Let $H = \{\alpha \in S_5 : \alpha(2) = 2\}$. Show that $H$ is a subgroup of $S_5$. Is $H$ normal in $S_5$?

Used in assessing outcome 4.


### 3 Summary

The following tables summarize the results of the various questions. The averages are weighted averages: exemplary has weight 2, acceptable has weight 1, and unacceptable has weight 0.

#### Outcome 1

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