Mathematics Major Assessment 2017

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1 Program Learning Outcomes

Students completing a degree in mathematics are expected to acquire the ability and skills to:

1. Use techniques of differentiation and integration of one and several variables;
2. Use differentiation and integration to solve problems in mathematics and other disciplines;
3. Solve and understand linear systems;
4. Give direct proofs, proofs by contradiction, and proofs by induction; formulate definitions and construct counterexamples;
5. Read mathematics without supervision; write mathematics with correct style, including typesetting;
6. Apply mathematics to problems in other disciplines; and
7. Use technology to solve mathematical problems.

2 Mode of Assessment

2.1 ETS Major Field Test for Mathematics

To assess the aforementioned Program Learning Outcomes, our graduating math majors took the ETS Major Field Test for Mathematics in May 2017.

2.2 Details about the exam

The exam is written by the Educational Testing Service, the same organization that writes the GRE and TOEFL. Over the last five years, this exam was taken by graduating math majors at a total of 305 institutions all across the United States. The total number of examinees in this time period is 8,997.

The exam has 50 multiple choice questions and covers topics most commonly offered as part of an undergraduate mathematics curriculum.

The content breakdown of the exam is as follows:

- **Calculus** (about 30%)
  Both single-variable and multivariable calculus.
- **Linear & Abstract Algebra** (about 30%)
  Matrices, linear transformations, eigenvalues, eigenvectors, vector spaces, systems of linear equations, elementary group/ring/field theory, elementary topics from number theory.
- **Additional Topics** (about 40%)
  Complex analysis, differential equations, discrete mathematics (including graph theory and combinatorics), foundations (including logic, proofs, sets, functions and relations), geometry, point-set topology, probability and statistics, and real analysis.

The exam questions are at three cognitive levels:

- **Routine** (about 55%)
  These questions cover definitions, questions with no more than a two-step reasoning process, or questions that require a standard technique that is practiced extensively in math courses at most institutions.
- **Non-routine** (about 25%)
  Includes questions that require an idea that is considered insightful, questions that require several steps of reasoning, and questions that require either the use of several definitions or a new definition that the student would not be expected to know. Some questions may require bringing techniques from two or more areas to bear on one problem.
- **Applied** (about 20%)
  This includes, for example, questions that are cast in real-world settings.

2.3 Sample Exam Questions (given on the ETS website)

MATHEMATICS TEST
SAMPLE QUESTIONS

The following questions illustrate the range of the test in terms of abilities measured, the disciplines covered, and the difficulty of the questions posed. They should not, however, be considered representative of the entire scope of the test in either content or difficulty. An answer key follows the questions.

1. A student is given an exam consisting of 8 essay questions divided into 4 groups of 2 questions each. The student is required to select a set of 6 questions to answer, including at least 1 question from each of the 4 groups. How many sets of questions satisfy this requirement?

   (A) 6
   (B) 24
   (C) 28
   (D) 48
   (E) 96

2. The function f is differentiable on the interval $(0, 4)$. If $f(1) - 1$ and $f(3) - 7$, then there is at least one c in $(1, 3)$ such that $f'(c) -$

   (A) -1
   (B) 0
   (C) 1
   (D) 2
   (E) 3

3. Let A and B be metric spaces, and let $f : A \to B$. Suppose that whenever X is an open set in B, the set $\{a \in A : f(a) \in X\}$ is closed in A. Which of the following must be true?

   I. f is injective.
   II. f is continuous.
   III. f is a homeomorphism.

   (A) None
   (B) II only
   (C) III only
   (D) I and III only
   (E) I, II, and III

4. In the xy-plane, the line tangent to the graph of $x^2 + xy + y^2 = 3$ at the point $(1, 1)$ has a slope of

   (A) -3
   (B) -1
   (C) 0
   (D) $\frac{1}{3}$
   (E) 1

5. Let $\mathbb{Z}$ be the ring of integers, and let R be a ring without identity. Let $S = \mathbb{Z} \times R$ be the ring with addition and multiplication defined by $(k, a) + (n, b) - (k + n, a + b)$ and $(k, a)(n, b) - (kn, kb + na + ab)$, where k and n are in $\mathbb{Z}$, and a and b are in R. Which of the following must be true about S ?

   I. S is a ring with identity.
   II. S has a subring isomorphic to R.
   III. S is an integral domain (it has no zero-divisors).

   (A) I only
   (B) II only
   (C) I and II only
   (D) I and III only
   (E) I, II, and III

$$\frac{dQ}{dt} = 6(5 - Q(t))$$
$$Q(0) = 0$$

6. The function $Q(t)$ satisfies the differential equation shown above. What is the value of t such that $Q(t) = 4$?

(A) $\frac{13}{3}$
(B) $\frac{\ln 5}{6}$
(C) $\frac{\ln 6}{5}$
(D) $30 - \frac{\ln 5}{6}$
(E) $30 + \frac{\ln 6}{5}$

7. What are the eigenvalues of $\begin{pmatrix} 6 & -3 \\ 1 & 2 \end{pmatrix}$?

(A) 1 and 15
(B) 2 and 6
(C) 3 and 5
(D) $\frac{5}{2} + \frac{i\sqrt{15}}{2}$ and $\frac{5}{2} - \frac{i\sqrt{15}}{2}$
(E) $4 + i$ and $4 - i$

8. What is the area of the portion of the surface $z = x^2 + y^2$ lying inside the cylinder $x^2 + y^2 = 4$ in xyz-space?

(A) 21π
(B) $\frac{21\pi}{2}$
(C) $\frac{\pi}{3}\left(17^{\frac{3}{2}}\right)$
(D) $\frac{\pi}{2}\left(17^{\frac{3}{2}} - 1\right)$
(E) $\frac{\pi}{6}\left(17^{\frac{3}{2}} - 1\right)$

9. $\int_{-1}^{1} \frac{e^x - e^{-x}}{e^{2x} + e^{-2x} + 2}\, dx =$

(A) 0
(B) 1
(C) $2e$
(D) $e + e^{-1}$
(E) $e^2 - e^{-2}$

10. If V_n is the real vector space of all n-tuples of real numbers for each $n > 1$, which of the following must be true?

I. Every basis of V_n contains exactly n vectors.
II. Every basis of V_n is an orthogonal set of vectors.
III. Every set of $n + 1$ vectors of V_n is a linearly dependent set.

(A) I only
(B) II only
(C) I and II
(D) I and III
(E) II and III

ANSWER KEY

1. B
2. E
3. B
4. B
5. C
6. B
7. C
8. E
9. A
10. D

3 Relationship between assessment and PLOs

This exam addresses Program Learning Outcomes 1, 2, 3, 4, 5, and 6.

- 30% percent of the exam problems cover *calculus knowledge*, which corresponds to Program Learning Outcomes 1 and 2.
- 30% of the exam problems cover *algebra knowledge*, which corresponds to Program Learning Outcomes 3 and 4.
- 25% of the exam problems are classified as *non-routine*, requiring several steps of reasoning or a new definition that the student would not be expected to know, which corresponds to Program Learning Outcomes 4 and 5.
- 20% of the exam problems are classified as *applied*, requiring the student to apply math to real-world settings. This corresponds to Program Learning Outcome 6.

4 Exam Results

Summary of Exam Results

	Number of Examinees	Mean Score	Standard Deviation
Nationwide	8,997	156.5	17.6
USF 2017	11	159	22
USF 2016	12	161	20

Notes: The scale range for the total score is 120 – 200. Nationwide data includes seniors from 305 different domestic institutions who tested between September 2012 through June 2017.

Performance by Question Type

	Calculus Questions	Algebra Questions	Applied Questions	Routine Questions	Non-routine Questions
Nationwide	30.6	33.1	34.5	31.1	26.2
USF 2017	30	35	38	32	30
USF 2016	30	45	33	38	29

Notes: These numbers represent the **mean percent correct** for each question type. Nationwide data includes seniors from domestic institutions who tested between September 2012 through June 2017, a total of 8,997 examinees among 305 institutions.

5 Observations

From the data in the previous section, it is clear that USF math majors performed at or above the national average. As a group, our students' mean score was 159, compared to the national mean of 156.5 – a negligible difference. If we break down our students' performance by question type, our students again performed at or above the national means.

We expect that over time we will observe trends in our test results. However, with only two years of data thus far, it is not yet possible to make any strong generalizations.

6 Assessment plans for next year

We plan to administer the exam again to our graduating math majors in May 2018.